KU LEUVEN
FACULTY OF ECONOMICS AND BUSINESS

Bayesian Poisson log-bilinear models for mortality projections with multiple populations - Online Appendix

Katrien Antonio, Anastasios Bardoutsos and Wilbert Ouburg

AFI_15104

Bayesian Poisson log-bilinear models for mortality projections with multiple populations

Online Appendix

Katrien Antonio[1,2], Anastasios Bardoutsos *[1], and Wilbert Ouburg[2,3]

[1]Faculty of Economics and Business, KU Leuven, Belgium.
[2]Faculty of Economics and Business, University of Amsterdam, The Netherlands.
[3]Delta Lloyd Groep, The Netherlands

June 5, 2015

Abstract

In this paper we present a detailed outline of the posterior distributions for the LL model, as described by Antonio et al. (2015). Moreover, we illustrate the convergence of the Markov Chain Monte Carlo ([MCMC]) updating scheme used by Antonio et al. (2015).

*Corresponding author. E-mail address: anastasios.bardoutsos@kuleuven.be

The authors gratefully acknowledge financial support from CAREFIN Bocconi Centre for Applied Research in Finance.

A Posterior distributions for LL model

We approach the LL model in two steps. We first sample the common parameters from

$$f\left(\boldsymbol{A},\boldsymbol{B},\boldsymbol{K}|\boldsymbol{D},\boldsymbol{E}\right) \propto f\left(\boldsymbol{D},\boldsymbol{E}|\boldsymbol{A},\boldsymbol{B},\boldsymbol{K}\right)\cdot f\left(\boldsymbol{A},\boldsymbol{B},\boldsymbol{K}\right). \tag{1}$$

Afterwards we sample the country specific parameters from

$$f\left(\boldsymbol{\alpha}^{(i)},\boldsymbol{\beta}^{(i)},\boldsymbol{\kappa}^{(i)}\middle|\boldsymbol{D}^{(i)},\boldsymbol{E}^{(i)},\boldsymbol{A},\boldsymbol{B},\boldsymbol{K}\right) \propto f\left(\boldsymbol{D}^{(i)},\boldsymbol{E}^{(i)},\boldsymbol{A},\boldsymbol{B},\boldsymbol{K}|\boldsymbol{\alpha}^{(i)},\boldsymbol{\beta}^{(i)},\boldsymbol{\kappa}^{(i)}\right)\cdot f\left(\boldsymbol{\alpha}^{(i)},\boldsymbol{\beta}^{(i)},\boldsymbol{\kappa}^{(i)}\right),$$

independently for each $i = 1, 2, \ldots, p$, given A_x, B_x and K_t. In both cases we use the Gibbs sampling technique. However, when the full conditional distribution is not explicitly available we use the Metropolis-Hastings ([MH]) algorithm. We introduce the following sets of data, latent effects and parameters

$$\boldsymbol{\theta} := \left(\boldsymbol{D},\boldsymbol{E},\boldsymbol{A},\boldsymbol{B},\boldsymbol{K},\boldsymbol{\sigma}_B^2,\gamma,\rho,\boldsymbol{\sigma}_K^2\right) \text{ and } \boldsymbol{\theta}_i := \left(\boldsymbol{D}^{(i)},\boldsymbol{E}^{(i)}\boldsymbol{A},\boldsymbol{B},\boldsymbol{K},\boldsymbol{\alpha}^{(i)},\boldsymbol{\beta}^{(i)},\boldsymbol{\kappa}^{(i)},\sigma^2_{\beta^{(i)}},\rho_{(i)},\sigma^2_{\kappa^{(i)}}\right),$$

for $i = 1, 2, \ldots, p$. Moreover, denote the vector with the sums of deaths and exposures as

$$\begin{aligned}
\boldsymbol{D}^{(\bullet)}_{\cdot,t} &= \left(\sum_{i=1}^p D^{(i)}_{x_{min},t},\ \ldots,\ \sum_{i=1}^p D^{(i)}_{x_{max},t}\right) \text{ and } \boldsymbol{D}^{(\bullet)}_{x,\cdot} = \left(\sum_{i=1}^p D^{(i)}_{x,t_{min}},\ \ldots,\ \sum_{i=1}^p D^{(i)}_{x,t_{max}}\right),\\
\boldsymbol{E}^{(\bullet)}_{\cdot,t} &= \left(\sum_{i=1}^p E^{(i)}_{x_{min},t},\ \ldots,\ \sum_{i=1}^p E^{(i)}_{x_{max},t}\right) \text{ and } \boldsymbol{E}^{(\bullet)}_{x,\cdot} = \left(\sum_{i=1}^p E^{(i)}_{x,t_{min}},\ \ldots,\ \sum_{i=1}^p E^{(i)}_{x,t_{max}}\right).
\end{aligned}$$

A.1 Metropolis-Hastings for κ's and K

For the common period effect K_t the proportional conditional probability density function for $t = t_{min}, \ldots, t_{max}$ is,

$$\begin{aligned}
f\left(K_t|\boldsymbol{\theta}\setminus\{K_t\}\right) &= f\left(K_t\middle|\boldsymbol{D},\boldsymbol{E},\boldsymbol{K}_{-t},\boldsymbol{A},\boldsymbol{B},\gamma,\rho,\sigma_K^2\right)\\
&\propto f\left(\boldsymbol{D}^{(\bullet)}_{\cdot,t},\boldsymbol{E}^{(\bullet)}_{\cdot,t}\middle|K_t,\boldsymbol{A},\boldsymbol{B}\right)\cdot f\left(K_t\middle|\boldsymbol{K}_{-t},\gamma,\rho,\sigma_K^2\right),
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
f\left(\boldsymbol{D}^{(\bullet)}_{\cdot,t},\boldsymbol{E}^{(\bullet)}_{\cdot,t}\middle|K_t,\boldsymbol{A},\boldsymbol{B}\right) \propto \prod_{x=x_{\min}}^{x_{\max}} &\exp\left(-\left(\sum_{i=1}^p E^{(i)}_{x,t}\right)\cdot\exp\left(A_x+B_xK_t\right)\right)\\
&\times\exp\left(B_xK_t\left(\sum_{i=1}^p D^{(i)}_{x,t}\right)\right).
\end{aligned} \tag{3}$$

We use Metropolis-Hastings to update K_t. Assume that we have $K_s^{[l]}$ for all $s < t$ at iteration $[l]$. Then we proceed as follows:

1. Generate a candidate K_t^* from a normal distribution with mean $K_t^{[l]}$ and variance $s^2_{K_t}$. We start with $s^2_{K_t} = 1$ and we update this value every 100 simulations to ensure that the acceptance probability is in the interval $[20\%, 50\%]$. For more details see Czado et al. (2005).

2. We use (2) to calculate the acceptance probability as follows:

$$\psi(K_t^{[l]}, K_t^*) = \min\left(1, \frac{f\left(K_t^* \middle| \boldsymbol{K}_{-t}^{[l]}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{D}, \sigma_K^2, \rho, \boldsymbol{\gamma}\right)}{f\left(K_t^{[l]} \middle| \boldsymbol{K}_{-t}^{[l]}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{D}, \sigma_K^2, \rho, \boldsymbol{\gamma}\right)}\right). \tag{4}$$

3. Generate a realization u from the Uniform$(0,1)$ distribution. If $u \leq \psi(K_t^{[l]}, K_t^*)$ we accept the candidate, i.e. $K_t^{[l+1]} = K_t^*$. If $u > \psi(K_t^{[l]}, K_t^*)$ we reject the candidate, so $K_t^{[l+1]} = K_t^{[l]}$.

4. Apply the constraint $\sum_{t=t_{min}}^{t_{max}} K_t = 0$.

The proportional conditional density function for $\kappa_t^{(i)}$ is obtained is a similar way:

$$f\left(\kappa_t^{(i)} \middle| \boldsymbol{\theta}_i \setminus \left\{\kappa_t^{(i)}\right\}\right) \propto f\left(\boldsymbol{D}_{.,t}^{(i)}, \boldsymbol{E}_{.,t}^{(i)} \middle| \kappa_t^{(i)}, \boldsymbol{\alpha}^{(i)}, \boldsymbol{\beta}^{(i)}\right) \cdot f\left(\kappa_t^{(i)} \middle| \boldsymbol{\kappa}_{-t}^{(i)}, \rho_{(i)}, \sigma_{\kappa^{(i)}}^2\right), \tag{5}$$

where

$$f\left(\boldsymbol{D}_{.,t}^{(i)}, \boldsymbol{E}_{.,t}^{(i)} \middle| \kappa_t^{(i)}, \boldsymbol{\alpha}^{(i)}, \boldsymbol{\beta}^{(i)}\right) \propto \prod_{x=x_{\min}}^{x_{\max}} \exp\left(-\tilde{E}_{x,t}^{(i)} \exp\left(\alpha_x^{(i)} + \beta_x^{(i)}\kappa_t^{(i)}\right)\right) \cdot \exp\left(\beta_x^{(i)}\kappa_t^{(i)} D_{x,t}^{(i)}\right), \tag{6}$$

with $\tilde{E}_{x,t}^{(i)} = E_{x,t}^{(i)} \cdot \exp\left(A_x + B_x K_t\right)$. We update the population specific time effect $\kappa_t^{(i)}$ in the same way for each population i. After each updating iteration we apply the constraint $\sum_{t=t_{min}}^{t_{max}} \kappa_t^{(i)} = 0$ for population i.

A.2 Metropolis-Hastings for β's and B

For the period effect B_x the proportional conditional probability density function for $x = x_{min}, \ldots, x_{max}$ is:

$$\begin{aligned} f\left(B_x \middle| \boldsymbol{\theta} \setminus \{B_x\}\right) &= f\left(B_x \middle| \boldsymbol{D}, \boldsymbol{E}, \boldsymbol{B}_{-x}, \boldsymbol{A}, \boldsymbol{B}, \sigma_B^2\right) \\ &\propto f\left(\boldsymbol{D}_{x,.}^{(\bullet)}, \boldsymbol{E}_{x,.}^{(\bullet)} \middle| B_x, \boldsymbol{A}, \boldsymbol{K}\right) \cdot f\left(B_x \middle| \boldsymbol{B}_{-x}, \sigma_B^2\right), \end{aligned} \tag{7}$$

where

$$\begin{aligned} f\left(\boldsymbol{D}_{x,.}^{(\bullet)}, \boldsymbol{E}_{x,.}^{(\bullet)} \middle| B_x, \boldsymbol{A}, \boldsymbol{K}\right) \propto \prod_{t=t_{\min}}^{t_{\max}} &\exp\left(-\left(\sum_{i=1}^{p} E_{x,t}^{(i)}\right) \exp\left(A_x + B_x K_t\right)\right) \\ &\times \exp\left(B_x K_t \left(\sum_{i=1}^{p} D_{x,t}^{(i)}\right)\right). \end{aligned} \tag{8}$$

We use Metropolis-Hastings to update B_x. Assume that we have $B_s^{[l]}$ for all $s < t$ at iteration $[l]$. We proceed as follows:

1. Generate a candidate B_x^* from a normal distribution with mean $B_x^{[l]}$ and variance $s_{B_x}^2$. We start with $s_{B_x}^2 = 1$ and we update this value every 100 simulations to ensure that the acceptance probability is in the interval [20%, 50%]. For more details see Czado et al. (2005).

2. We use (7) to calculate the acceptance probability as follows:

$$\psi(B_x^{[l]}, B_x^*) = \min\left(1, \frac{f\left(B_x^* \middle| \boldsymbol{B}_{-x}^{[l]}, \boldsymbol{A}, \boldsymbol{K}, \boldsymbol{D}, \sigma_B^2\right)}{f\left(B_x^{[l]} \middle| \boldsymbol{B}_{-x}^{[l]}, \boldsymbol{A}, \boldsymbol{K}, \boldsymbol{D}, \sigma_B^2\right)}\right) \tag{9}$$

3. Generate a realization u from the Uniform$(0,1)$ distribution. If $u \leq \psi(B_x^{[l]}, B_x^*)$ we accept the candidate, i.e. $B_x^{[l+1]} = B_x^*$. If $u > \psi(B_x^{[l]}, B_x^*)$ we reject the candidate, so $B_x^{[l+1]} = B_x^{[l]}$.

4. Apply the constraint $\sum_{x=x_{min}}^{x_{max}} B_x = 1$.

In a similar way we obtain for $\beta_x^{(i)}$:

$$f\left(\beta_x^{(i)} \middle| \boldsymbol{\theta}_i \setminus \left\{\beta_x^{(i)}\right\}\right) \propto f\left(\boldsymbol{D}_{x,.}^{(i)}, \boldsymbol{E}_{x,.}^{(i)} \middle| \beta_x^{(i)}, \boldsymbol{\alpha}^{(i)}, \boldsymbol{\kappa}^{(i)}\right) \cdot f\left(\beta_x^{(i)} \middle| \boldsymbol{\beta}_{-x}^{(i)}, \sigma_{\beta^{(i)}}^2\right), \tag{10}$$

where

$$f\left(\boldsymbol{D}_{x,.}^{(i)}, \boldsymbol{E}_{x,.}^{(i)} \middle| \beta_x^{(i)}, \boldsymbol{\kappa}^{(i)}, \boldsymbol{\alpha}^{(i)}\right) \propto \prod_{t=t_{\min}}^{t_{\max}} \exp\left(-\tilde{E}_{x,t}^{(i)} \exp\left(\alpha_x^{(i)} + \beta_x^{(i)}\kappa_t^{(i)}\right)\right) \times \exp\left(\beta_x^{(i)}\kappa_t^{(i)}D_{x,t}^{(i)}\right), \tag{11}$$

with $\tilde{E}_{x,t}^{(i)} = E_{x,t}^{(i)} \cdot \exp\left(A_x + B_x K_t\right)$. The MH steps for the population specific age effect $\beta_x^{(i)}$ are similar for each population i. After each updating iteration we apply the constraint $\left\|\beta_x^{(i)}\right\|_2 = 1$ for population i, where $\|.\|_2$ denotes the Euclidean norm.

A.3 Gibbs sampling for α's and A

In this case we simulate directly from the posterior distribution because we choose a conjugate prior for the likelihood, namely

$$\mathcal{E}_x := \exp(A_x) \sim \mathcal{G}(a_x, b_x). \tag{12}$$

The posterior distribution of $\mathcal{E}_x$ is,

$$\begin{aligned} f\left(\mathcal{E}_x \middle| \boldsymbol{\theta} \setminus \{\mathcal{E}_x\}\right) = f\left(\mathcal{E}_x \middle| \boldsymbol{B}, \boldsymbol{K}, \boldsymbol{D}, \boldsymbol{E}\right) &\propto f\left(\boldsymbol{D}_{x,.}^{(\bullet)}, \boldsymbol{E}_{x,.}^{(\bullet)} \middle| \mathcal{E}_x, \boldsymbol{B}, \boldsymbol{K}\right) \cdot f\left(\mathcal{E}_x\right) \\ &\propto \exp\left(-\left(b_x + C_x\right)\mathcal{E}_x\right)\left(\mathcal{E}_x\right)^{a_x + \sum_{i=1}^p D_{x,\bullet}^{(i)} - 1}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} f\left(\boldsymbol{D}_{x,.}^{(\bullet)}, \boldsymbol{E}_{x,.}^{(\bullet)} \middle| \mathcal{E}_x, \boldsymbol{B}, \boldsymbol{K}\right) &\propto \prod_{t=t_{\min}}^{t_{\max}} \exp\left(-\left(\sum_{i=1}^p E_{x,t}^{(i)}\right)\exp\left(A_x + B_x K_t\right) + A_x \sum_{i=1}^p D_{x,t}^{(i)}\right) \\ &\propto \exp\left(-C_x \mathcal{E}_x\right)\left(\mathcal{E}_x\right)^{\sum_{i=1}^p D_{x,\bullet}^{(i)}}, \end{aligned} \tag{14}$$

with $C_x := \sum_{i=1}^p \sum_{t=t_{\min}}^{t_{\max}} \exp\left(B_x K_t\right) E_{x,t}^{(i)}$ and $D_{x,\bullet}^{(i)} := \sum_{x=x_{\min}}^{x_{\max}} D_{x,t}^{(i)}$. Hence the posterior distribution is a Gamma distribution, namely

$$\left(\mathcal{E}_x \middle| \boldsymbol{\theta} \setminus \{\mathcal{E}_x\}\right) \sim \mathcal{G}\left(a_x + \sum_{i=1}^p D_{x,\bullet}^{(i)}, b_x + C_x\right). \tag{15}$$

For the population specific $\alpha_x^{(i)}$ recall the prior distribution,

$$e_x^{(i)} := \exp\left(\alpha_x^{(i)}\right) \sim \mathcal{G}(a_x^{(i)}, b_x^{(i)}). \tag{16}$$

The posterior distribution of $e_x^{(i)}$ is

$$\begin{aligned} f\left(e_x^{(i)} \middle| \boldsymbol{\theta}_i \setminus \left\{e_x^{(i)}\right\}\right) &= f\left(e_x^{(i)} \middle| \boldsymbol{D}_{x,.}^{(i)}, \boldsymbol{E}_{x,.}^{(i)}, \boldsymbol{\beta}^{(i)}, \boldsymbol{\kappa}^{(i)}\right) \\ &\propto f\left(\boldsymbol{D}_{x,.}^{(i)}, \boldsymbol{E}_{x,.}^{(i)} | e_x^{(i)}, \boldsymbol{\beta}^{(i)}, \boldsymbol{\kappa}^{(i)}\right) \cdot f\left(e_x^{(i)}\right) \\ &\propto \exp\left(-\left(b_x^{(i)} + c_x^{(i)}\right) e_x^{(i)}\right) \left(e_x^{(i)}\right)^{a_x^{(i)} + D_{x,\bullet}^{(i)} - 1}, \end{aligned} \tag{17}$$

where $c_x^{(i)} := \sum_{t=t_{min}}^{t_{max}} E_{x,t}^{(i)} \exp\left(A_x + B_x K_t + \beta_x^{(i)} \kappa_t^{(i)}\right)$. The posterior distribution is therefore Gamma distributed, namely

$$\left(e_x^{(i)} \middle| \boldsymbol{\theta}_i \setminus \left\{e_x^{(i)}\right\}\right) \sim \mathcal{G}\left(a_x^{(i)} + D_{x,\bullet}^{(i)}, b_x^{(i)} + c_x^{(i)}\right). \tag{18}$$

B Convergence diagnostics

We run 20,000 iterations of the MCMC algorithm for the LC-2,t model. We remove an initial, burn-in period of 4,000 iterations, and collect information from the remaining iterations, using a thinning factor of 10. This leads to a final sample size of 1,600. For the LL model we first update the common parameters as described in Appendix A. This results in a sample of size 8000. After reaching convergence of the MCMC updating scheme for the common parameters, we simulate the population specific parameters from their posterior distributions, while replacing in each iteration the common parameters in the likelihood with a value sampled from their posterior distributions. After burn-in and thinning our final sample size is again 1,600. For both models, we perform the usual convergence checks. The trace plots listed below show good mixing properties. In the figures, the blue line represents the median of the 1,600 iterations kept after thinning and burn-in. The shaded gray area is the 95% pointwise credibility interval using these iterations.

B.1 Swedish male and female mortality with LC-2,t model

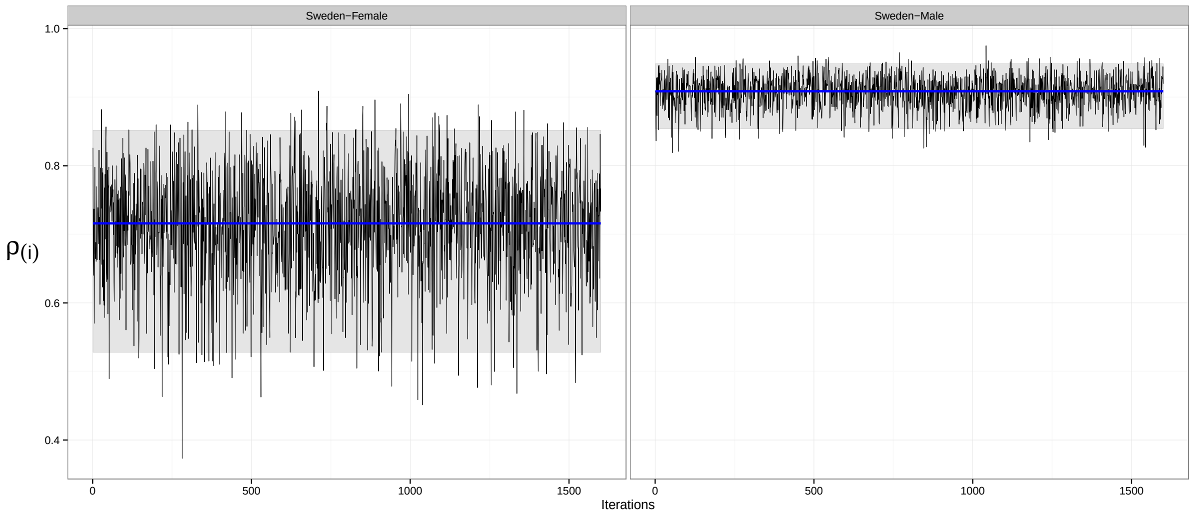


Figure 1: Model LC-2,t: trace plot for hyper-parameter $\rho_{(i)}$ for females (left) and males (right): Sweden, period 1950-2009 and ages 0-89.

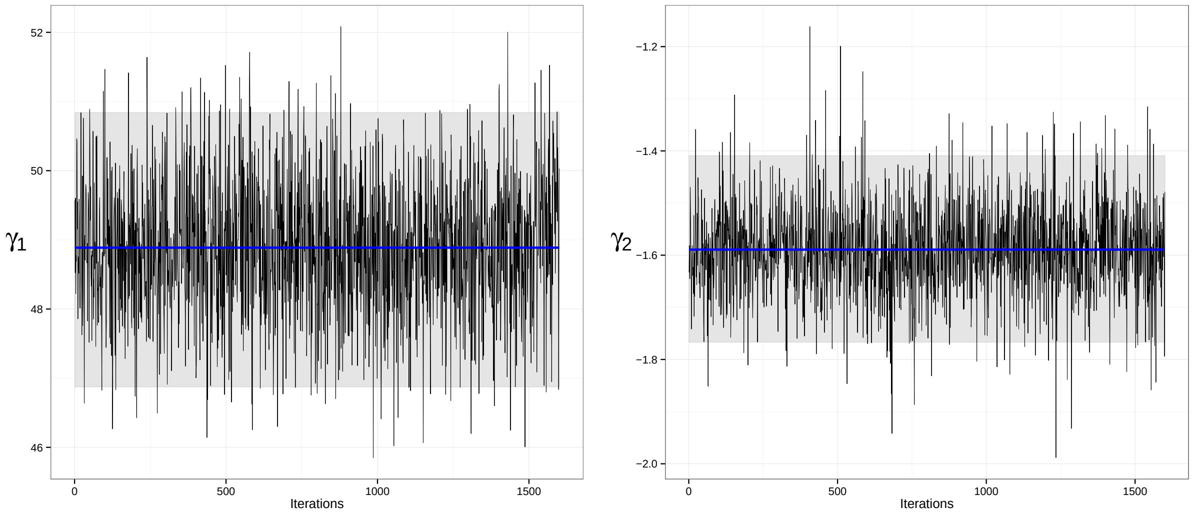


Figure 2: Model LC-2,t: trace plot for hyper-parameters γ_1 (left) and γ_2 (right) for the common trend: Sweden, period 1950-2009 and ages 0-89.

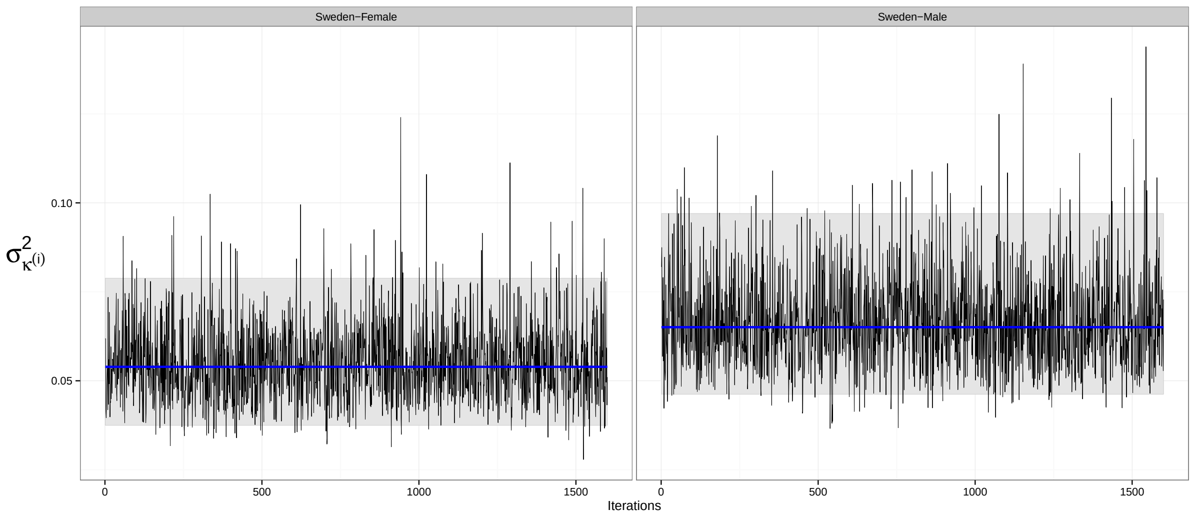
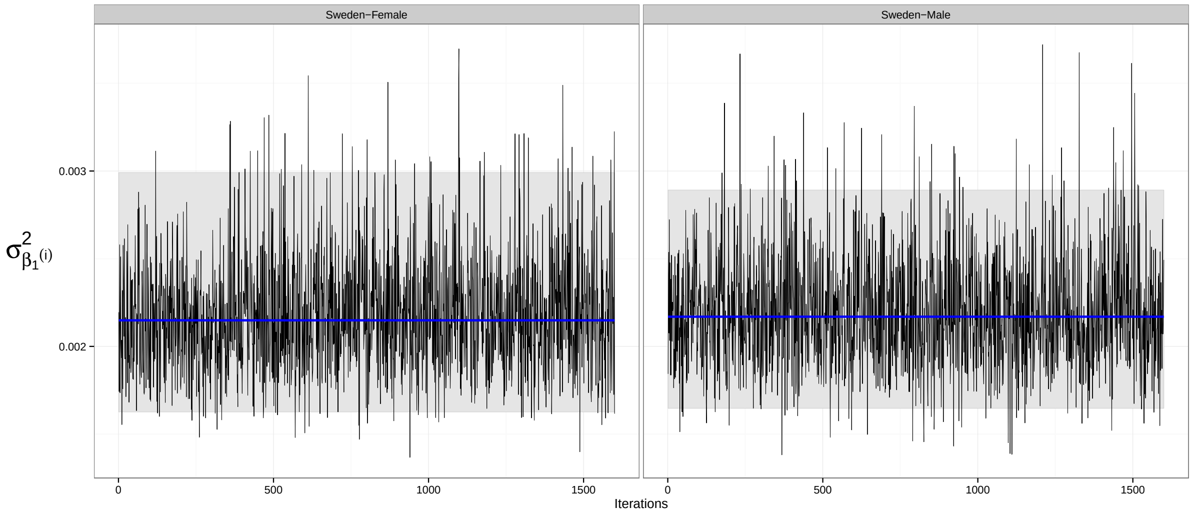
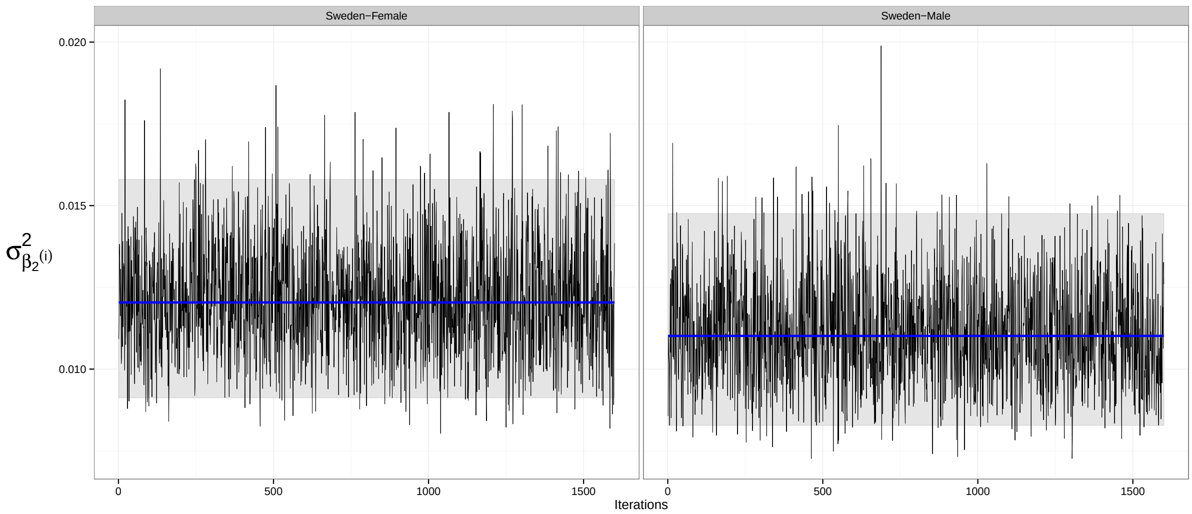


Figure 3: Model LC-2,t: trace plot for hyper-parameter $\sigma^2_{\kappa^{(i)}}$ for females (left) and males (right): Sweden, period 1950-2009 and ages 0-89.



(a) Model LC-2,t: trace plot for hyper-parameter $\sigma^2_{\beta_1^{(i)}}$ for females (left) and males (right): Sweden, period 1950-2009 and ages 0-89.



(b) Model LC-2,t: trace plot for hyper-parameter $\sigma^2_{\beta_2^{(i)}}$ for females (left) and males (right): Sweden, period 1950-2009 and ages 0-89.

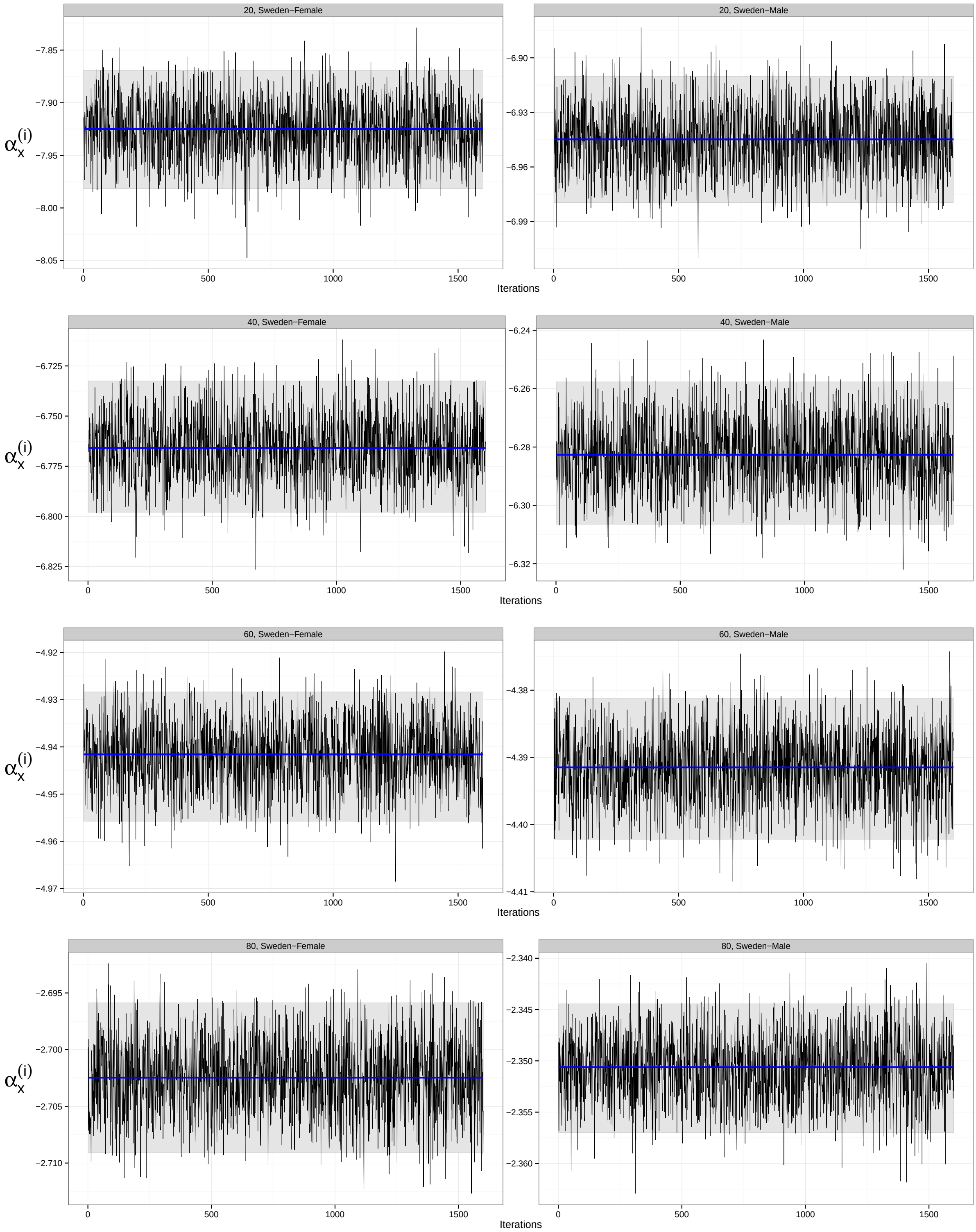


Figure 5: Model LC-2,t: trace plot for parameter $\alpha_x^{(i)}$ at ages 20, 40, 60 and 80 for females (left) and males (right): Sweden, period 1950-2009 and ages 0-89.

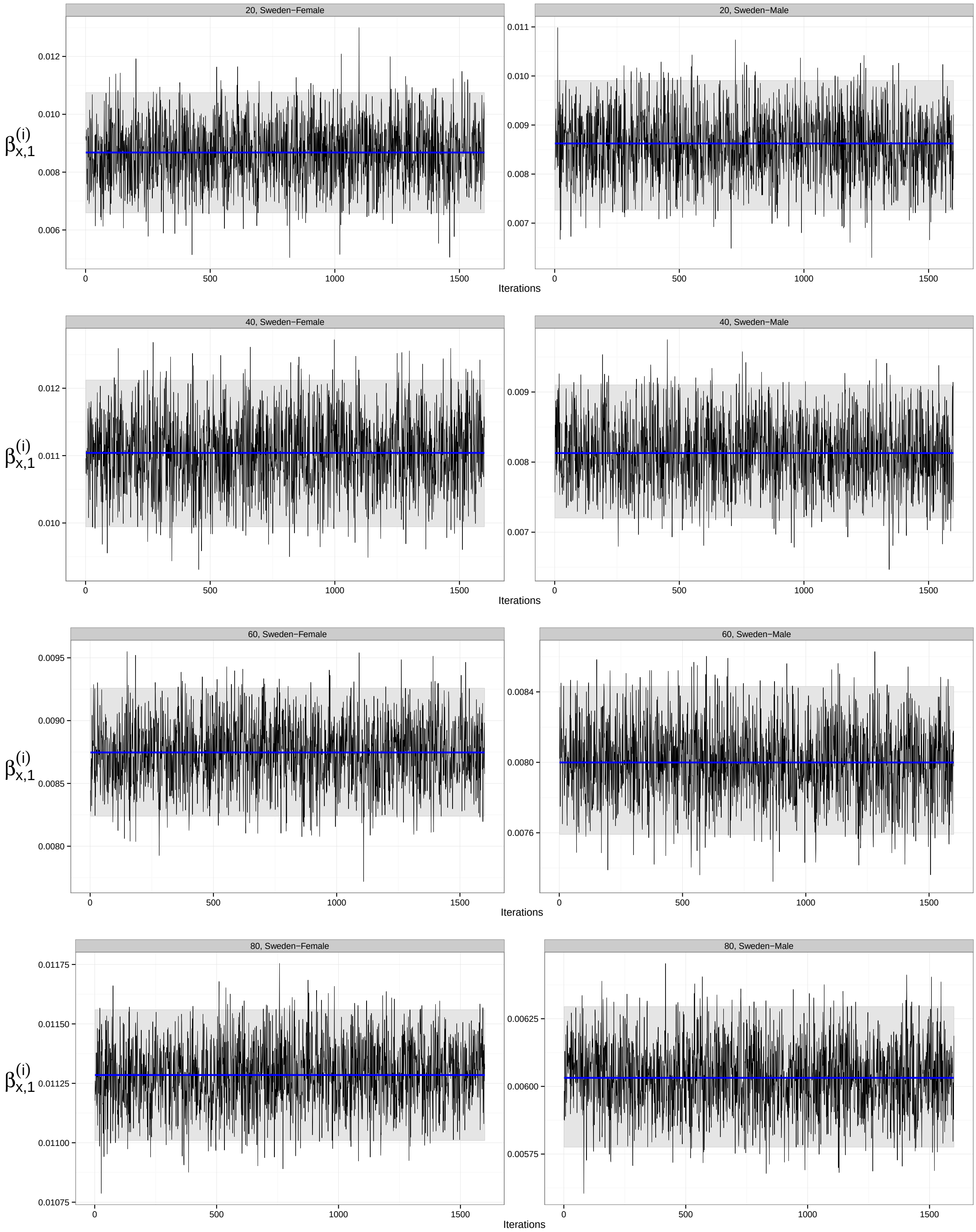


Figure 6: Model LC-2,t: trace plot for parameter $\beta_{x,1}^{(i)}$ at ages 20, 40, 60 and 80 for females (left) and males (right): Sweden, period 1950-2009 and ages 0-89.

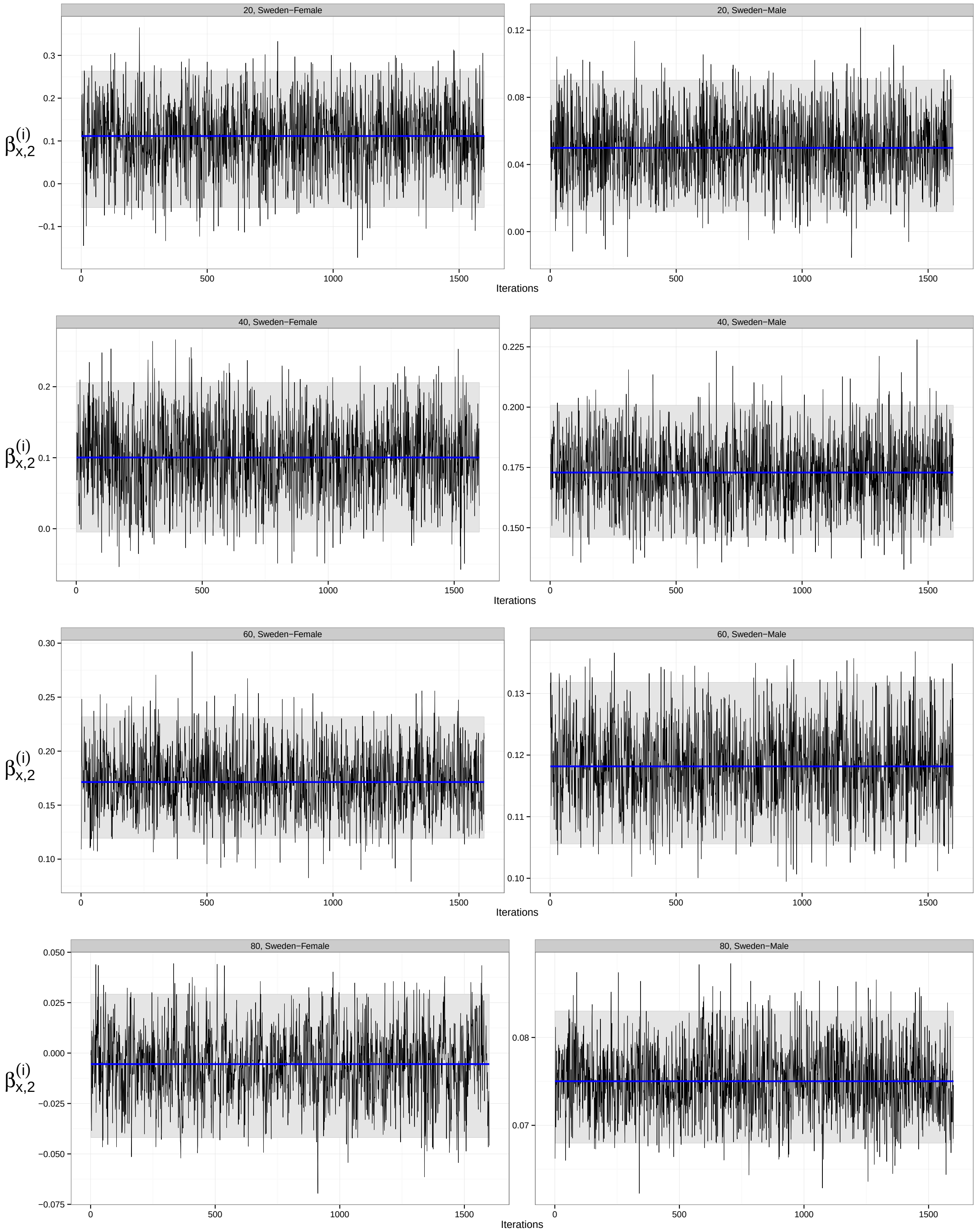


Figure 7: Model LC-2,t: trace plot for parameter $\beta_{x,2}^{(i)}$ at ages 20, 40, 60 and 80 for females (left) and males (right): Sweden, period 1950-2009 and ages 0-89.

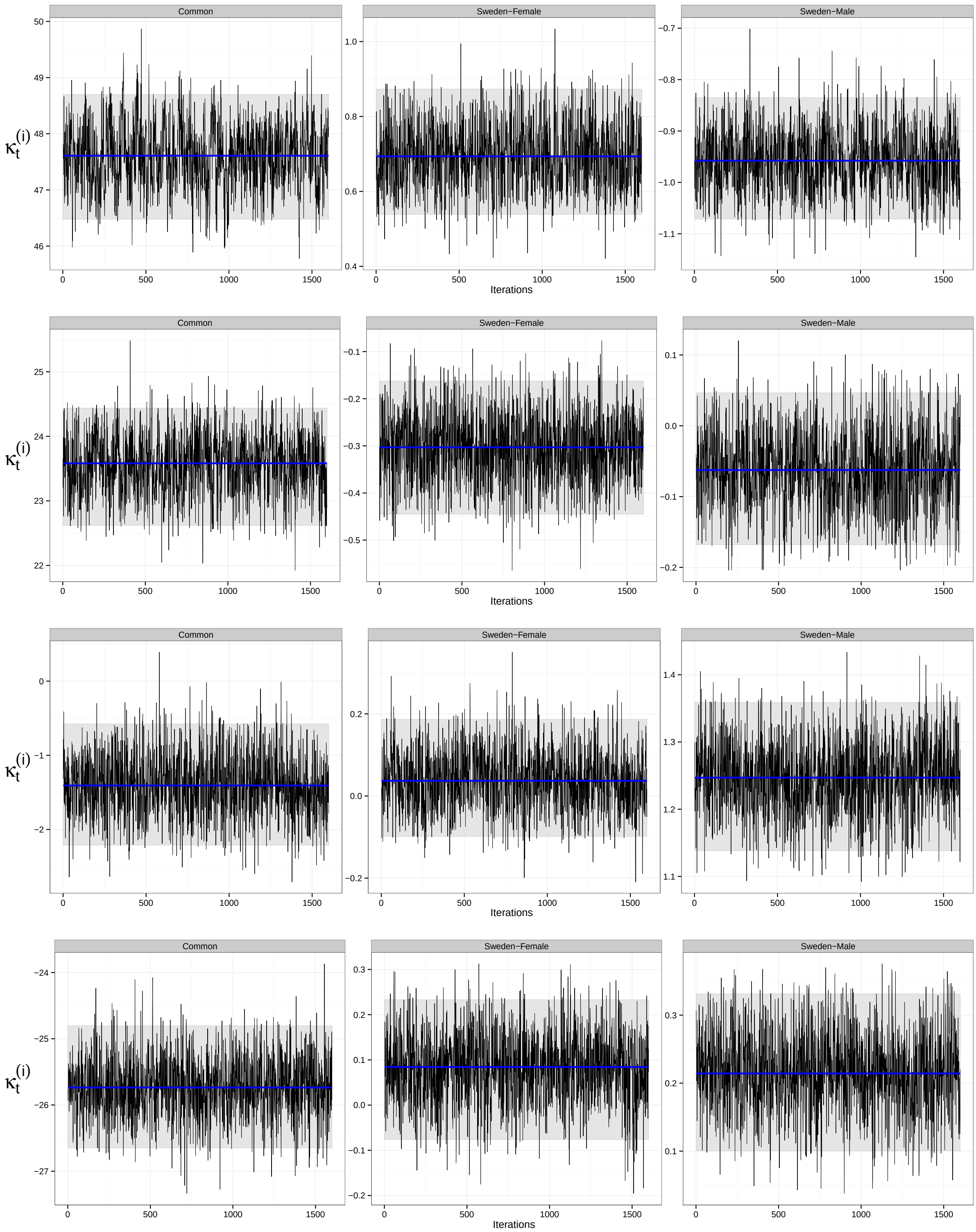


Figure 8: Model LC-2,t: trace plots for parameters K_t (left, with label 'Common') and $\kappa_t^{(i)}$ at years 1950, 1965, 1980 and 1995 for females (middle) and males (right): Sweden, period 1950-2009 and ages 0-89.

B.2 14 countries with LC-2,t model

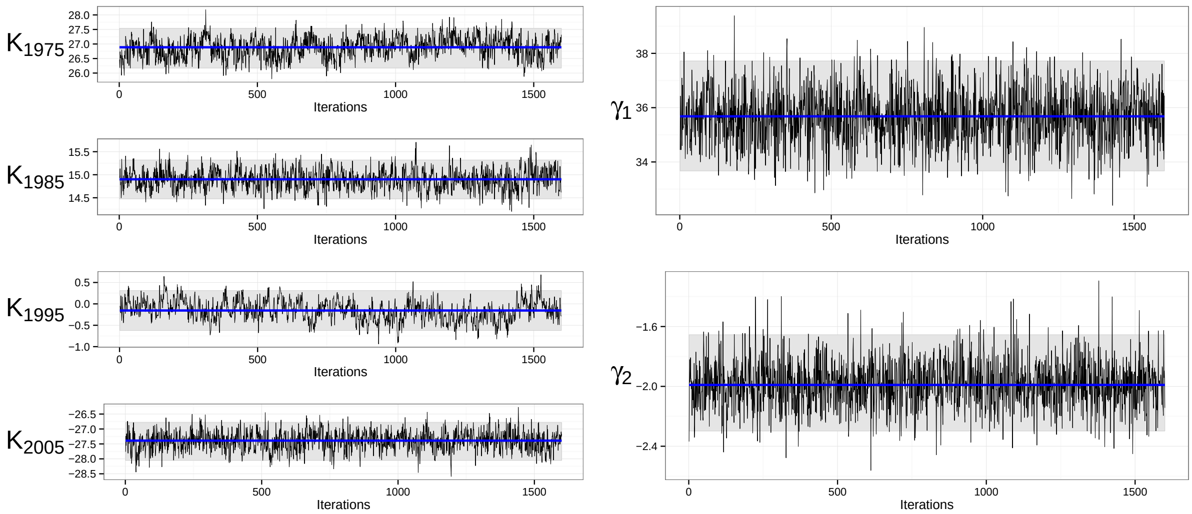


Figure 9: Model LC-2,t: trace plots for parameter K_t (left) for years 1975, 1985, 1995 and 2005 and hyper-parameters γ_1 (top right) and γ_2 (bottom right): female, period 1975-2009 and ages 0-89.

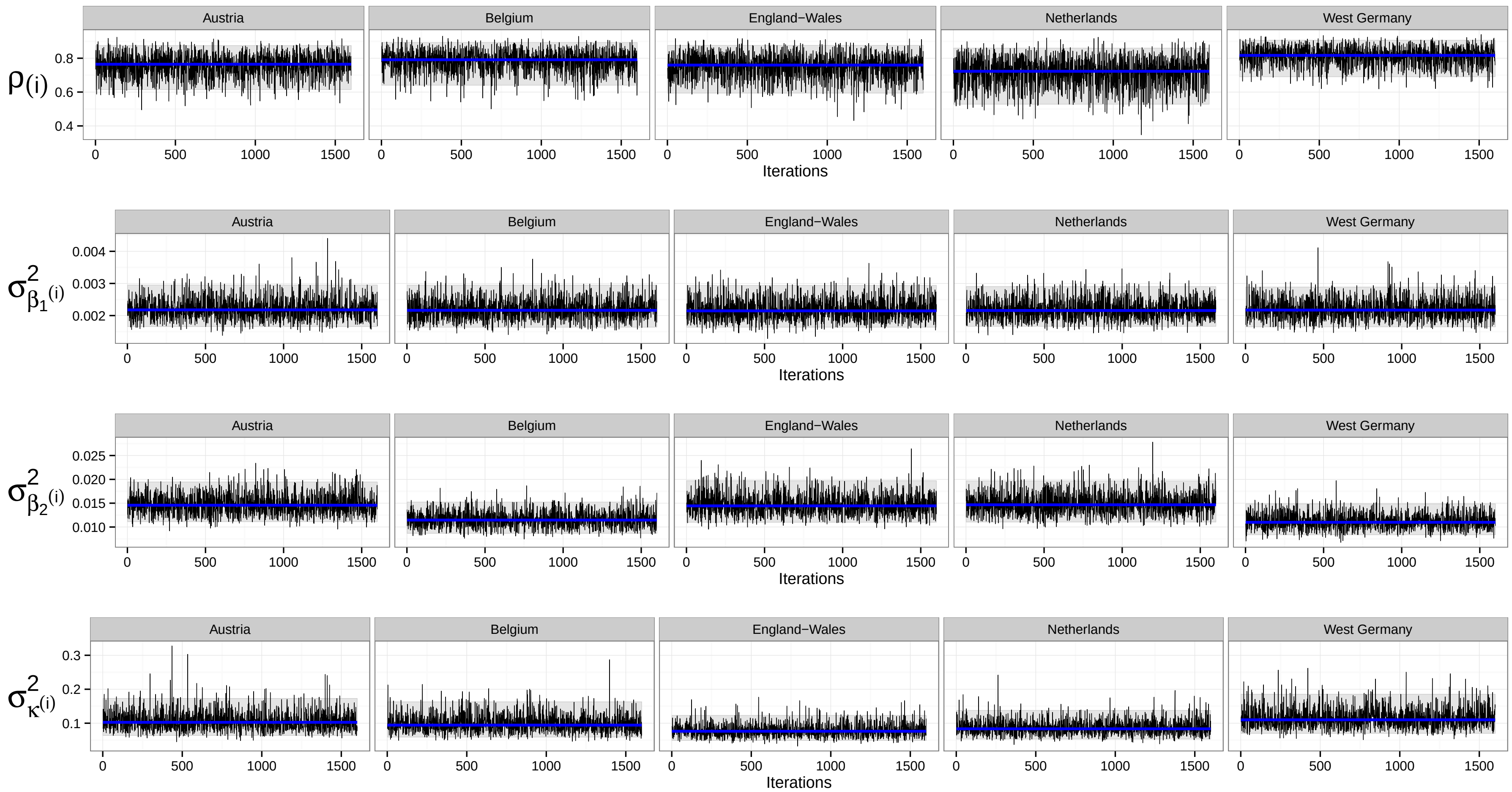


Figure 10: Model LC-2,t: trace plots for hyper-parameters $\rho_{(i)}$, $\sigma_{\beta_1}^{(i)}$, $\sigma_{\beta_2}^{(i)}$ and $\sigma_{\kappa}^{(i)}$ for Austria, Belgium, England-Wales, The Netherlands and West Germany: female, period 1975-2009 and ages 0-89.

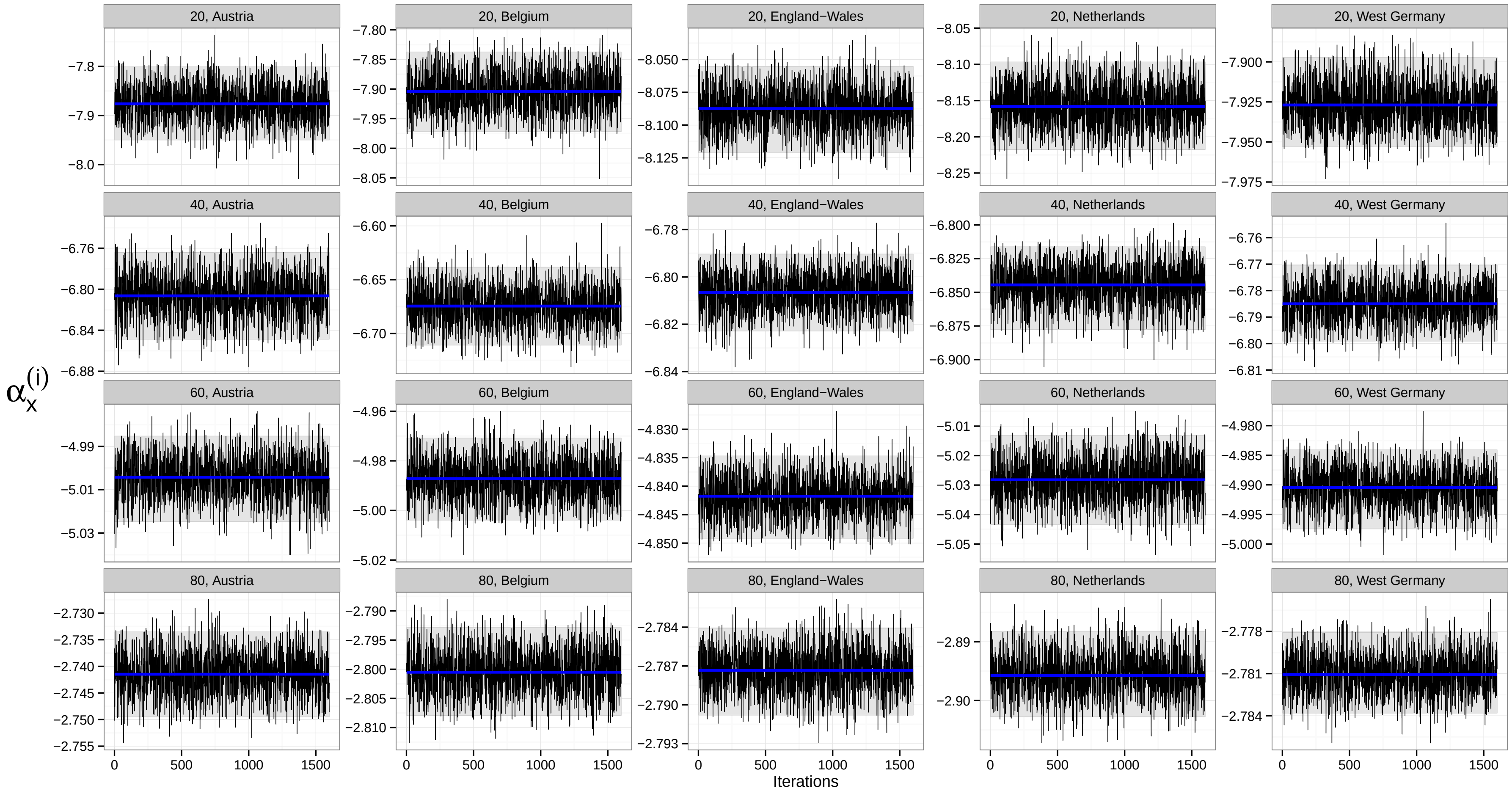


Figure 11: Model LC-2,t: trace plots for parameter $\alpha_x^{(i)}$ at ages 20, 40, 60 and 80 for Austria, Belgium, England-Wales, The Netherlands and West Germany: female, period 1975-2009 and ages 0-89.

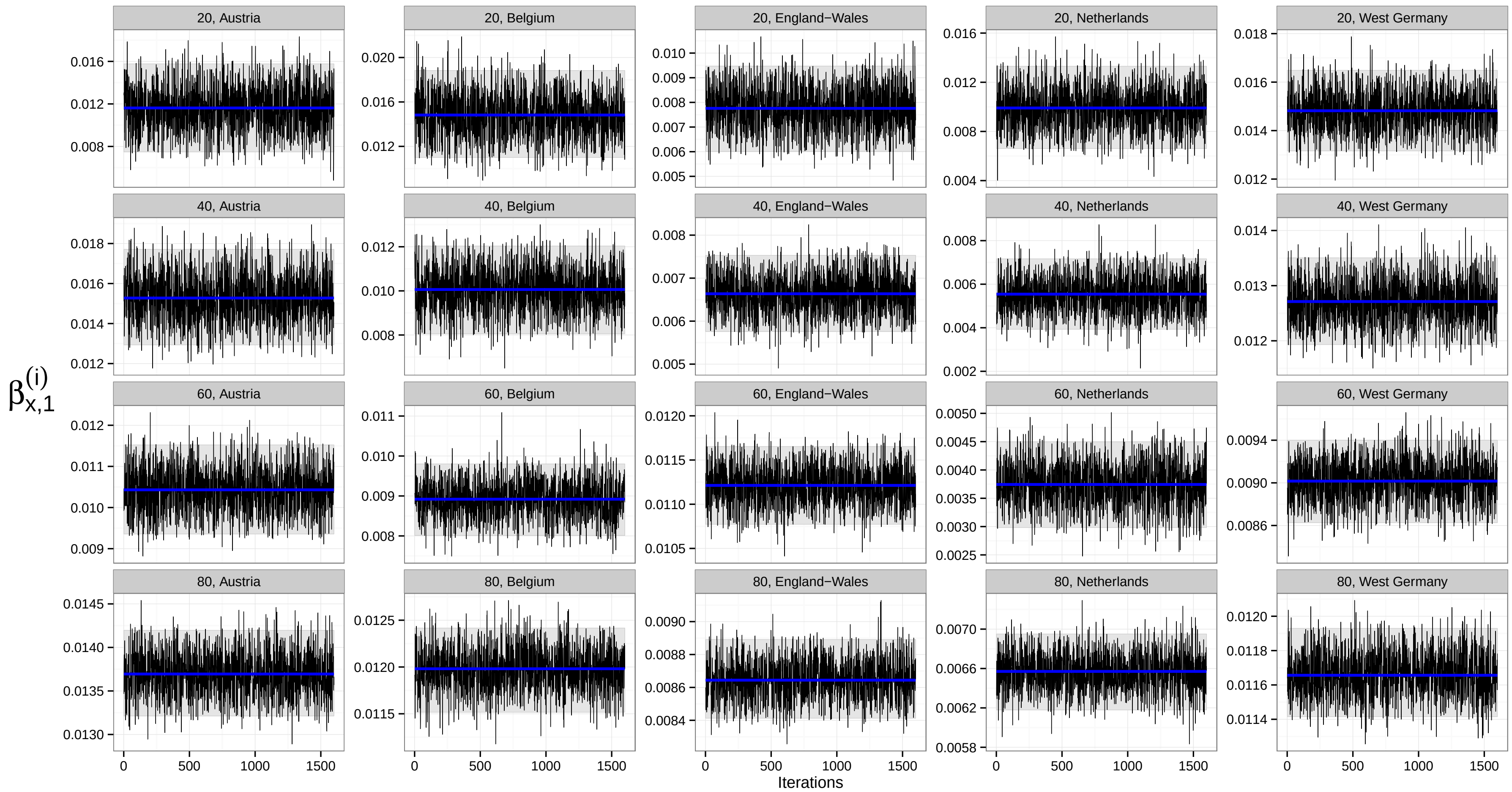


Figure 12: Model LC-2,t: trace plots for parameter $\beta_{x,1}^{(i)}$ at ages 20, 40, 60 and 80 for Austria, Belgium, England-Wales, The Netherlands and West Germany: female, period 1975-2009 and ages 0-89.

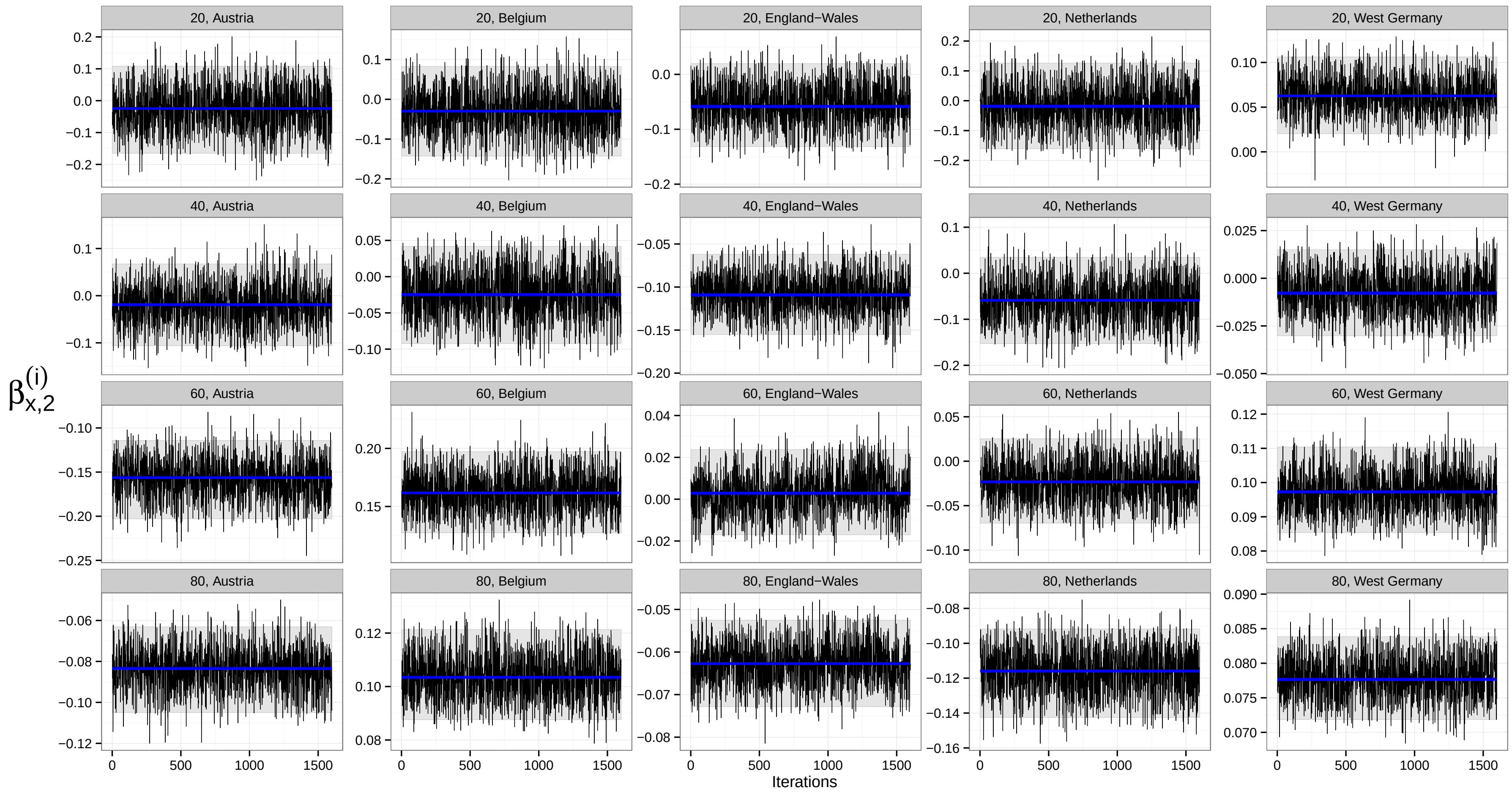


Figure 13: Model LC-2,t: trace plots for parameter $\beta^{(i)}_{x,2}$ at ages 20, 40, 60 and 80 for Austria, Belgium, England-Wales, The Netherlands and West Germany: female, period 1975-2009 and ages 0-89.

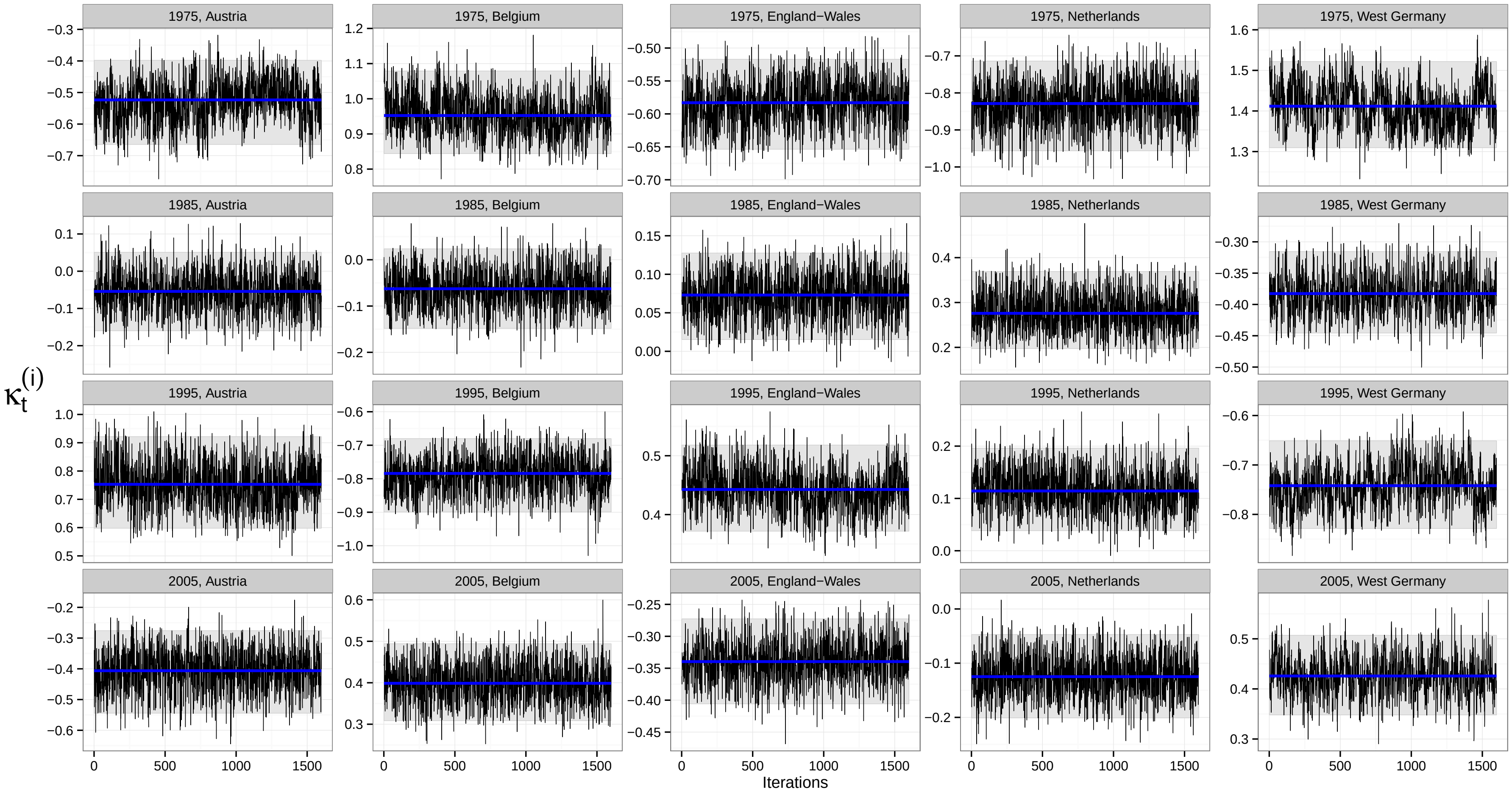


Figure 14: Model LC-2,t: trace plots for parameter $\kappa_t^{(i)}$ at years 1975, 1985, 1995 and 2005 for Austria, Belgium, England-Wales, The Netherlands and West Germany: female, period 1975-2009 and ages 0-89.

B.3 Swedish male and female mortality with LL model

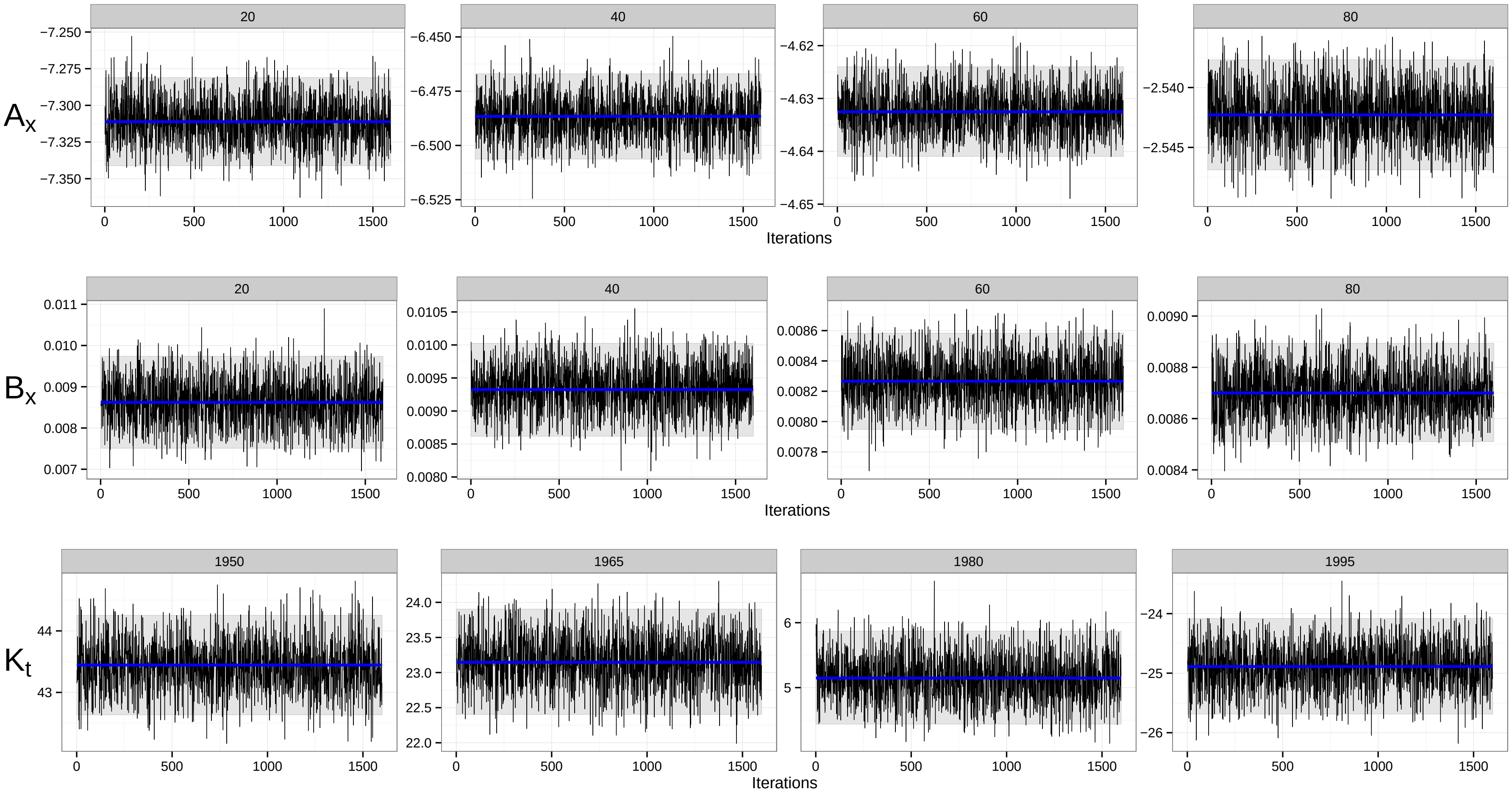


Figure 15: Model LL: trace plots for common parameters A_x, B_x at ages 20, 40, 60 and 80 and parameter K_t at years 1950, 1965, 1980 and 1995: Sweden, period 1950-2009 and ages 0-89.

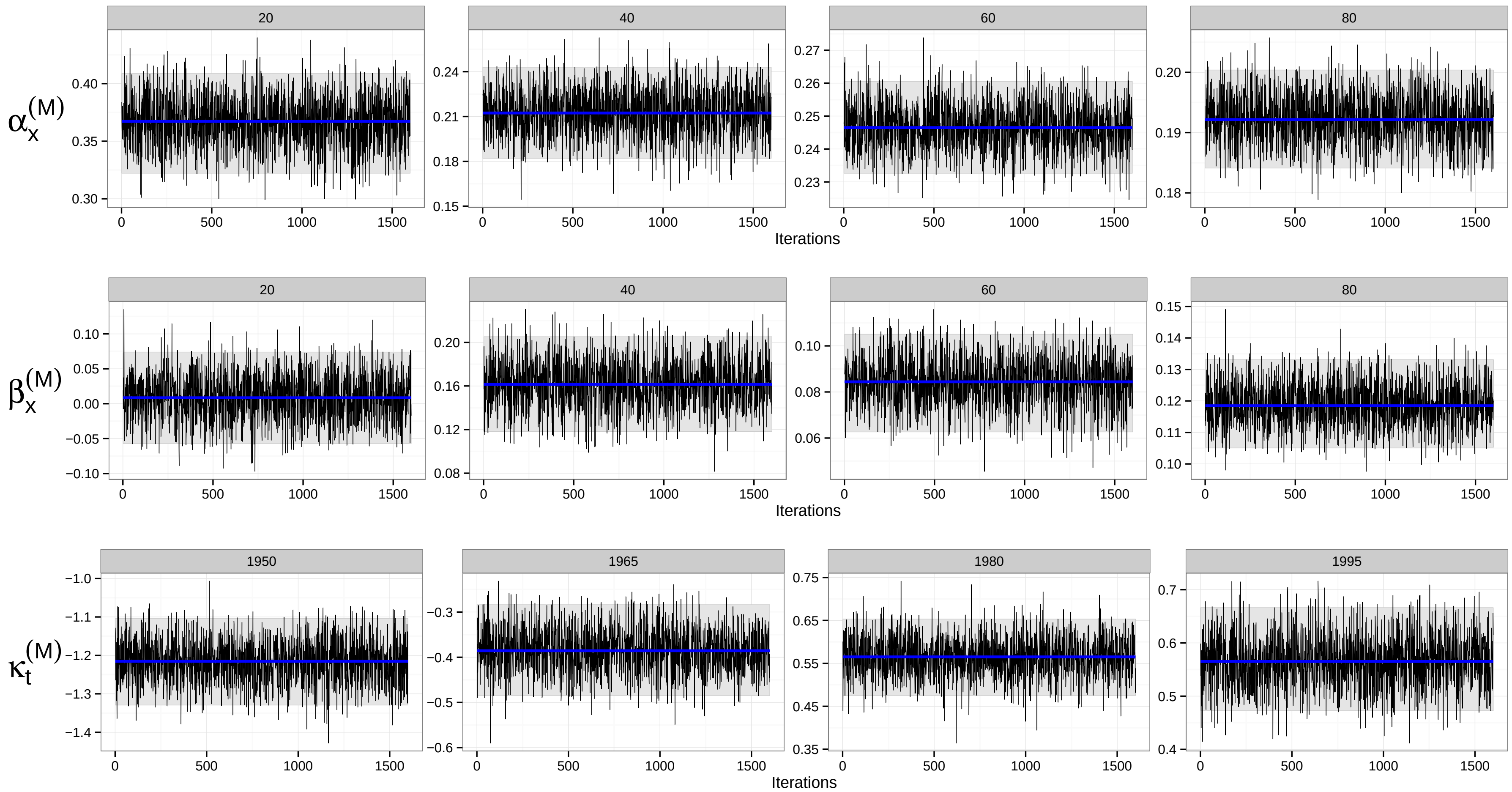


Figure 16: Model LL: trace plots for parameters $\alpha_x^{(M)}$, $\beta_x^{(M)}$ at ages 20, 40, 60 and 80 and parameter $\kappa_t^{(M)}$ at years 1950, 1965, 1980 and 1995 for Swedish males, period 1950-2009 and ages 0-89.

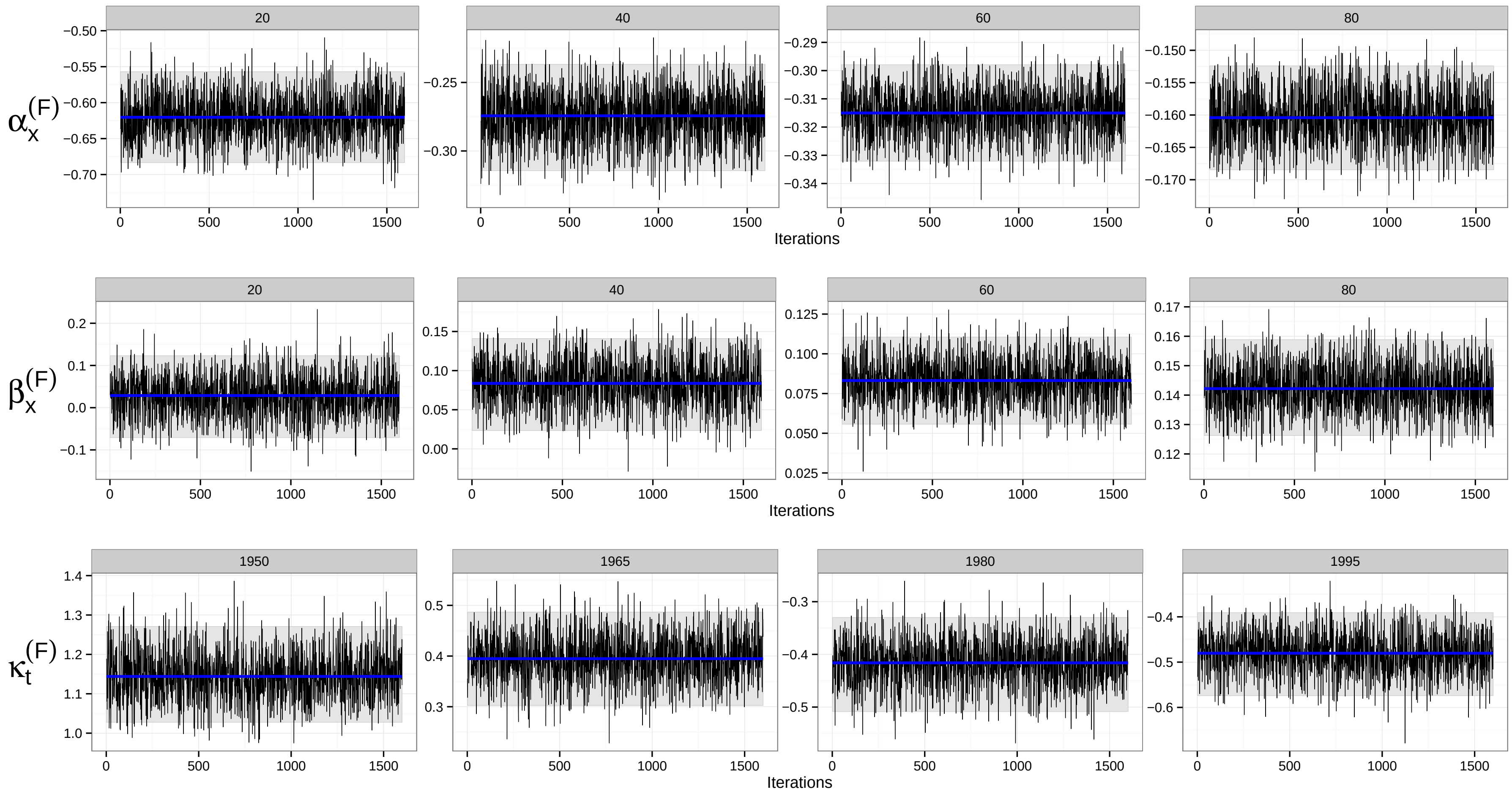


Figure 17: Model LL: trace plots for parameters $\alpha_x^{(F)}$, $\beta_x^{(F)}$ at ages 20, 40, 60 and 80 and parameter $\kappa_t^{(F)}$ at years 1950, 1965, 1980 and 1995 for Swedish females, period 1950-2009 and ages 0-89.

B.4 14 countries with LL model

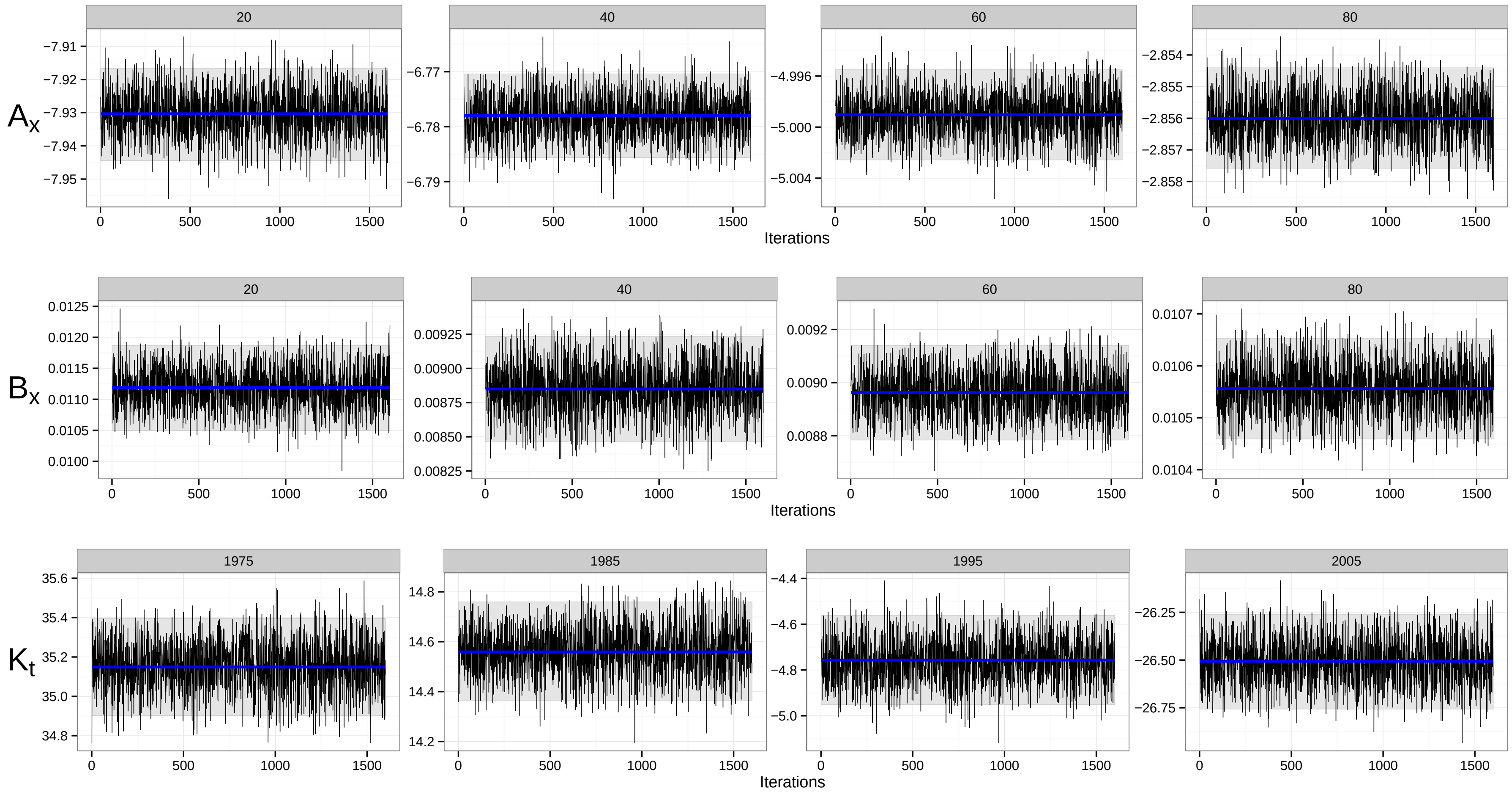


Figure 18: Model LL: trace plots for common parameters A_x, B_x at ages 20, 40, 60 and 80 and parameter K_t at years 1975, 1985, 1995 and 2005: female, period 1975-2009 and ages 0-89.

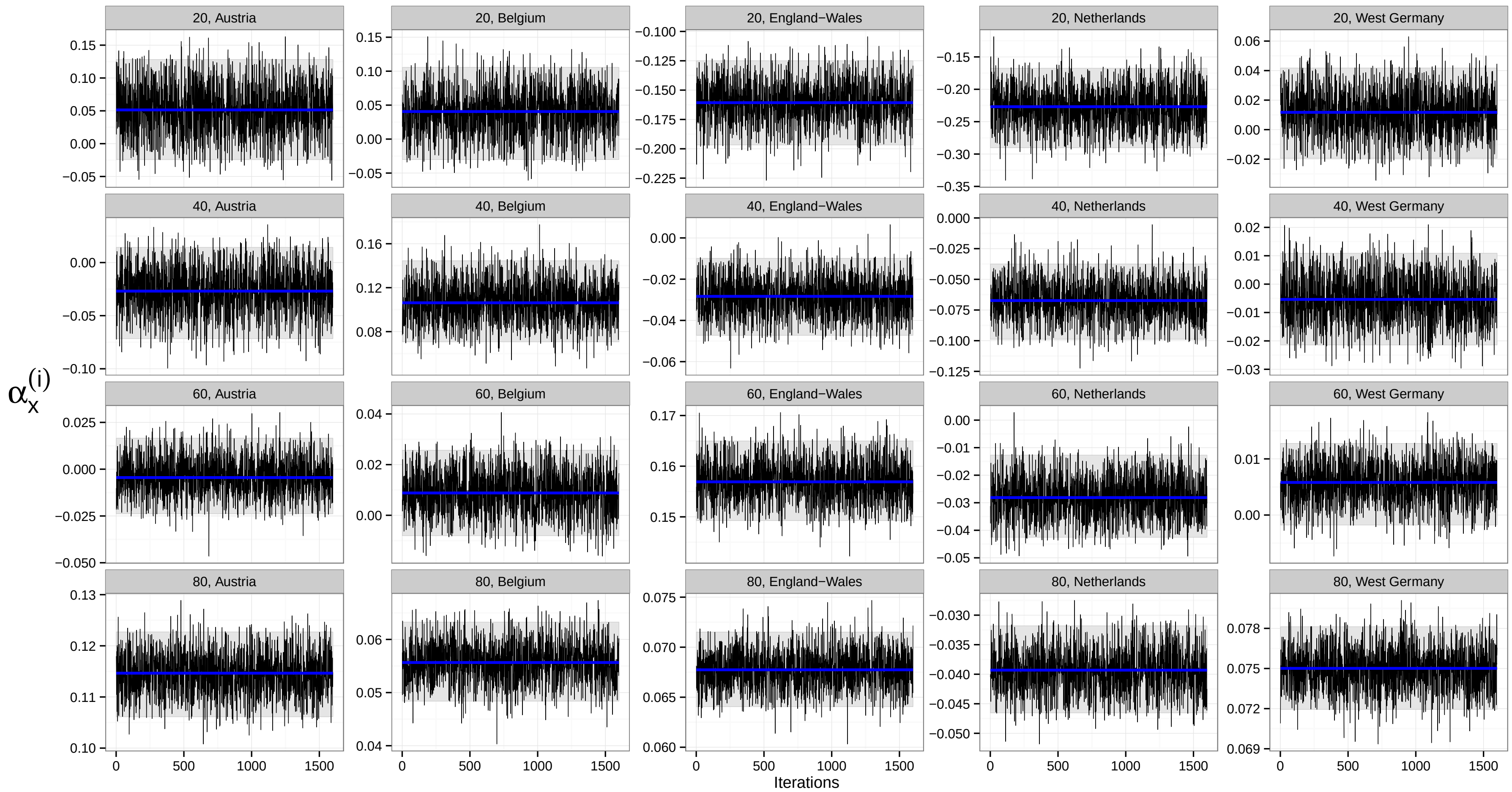


Figure 19: Model LL: trace plots for parameters $\alpha_x^{(i)}$ at ages 20, 40, 60 and 80: female, period 1975-2009 and ages 0-89.

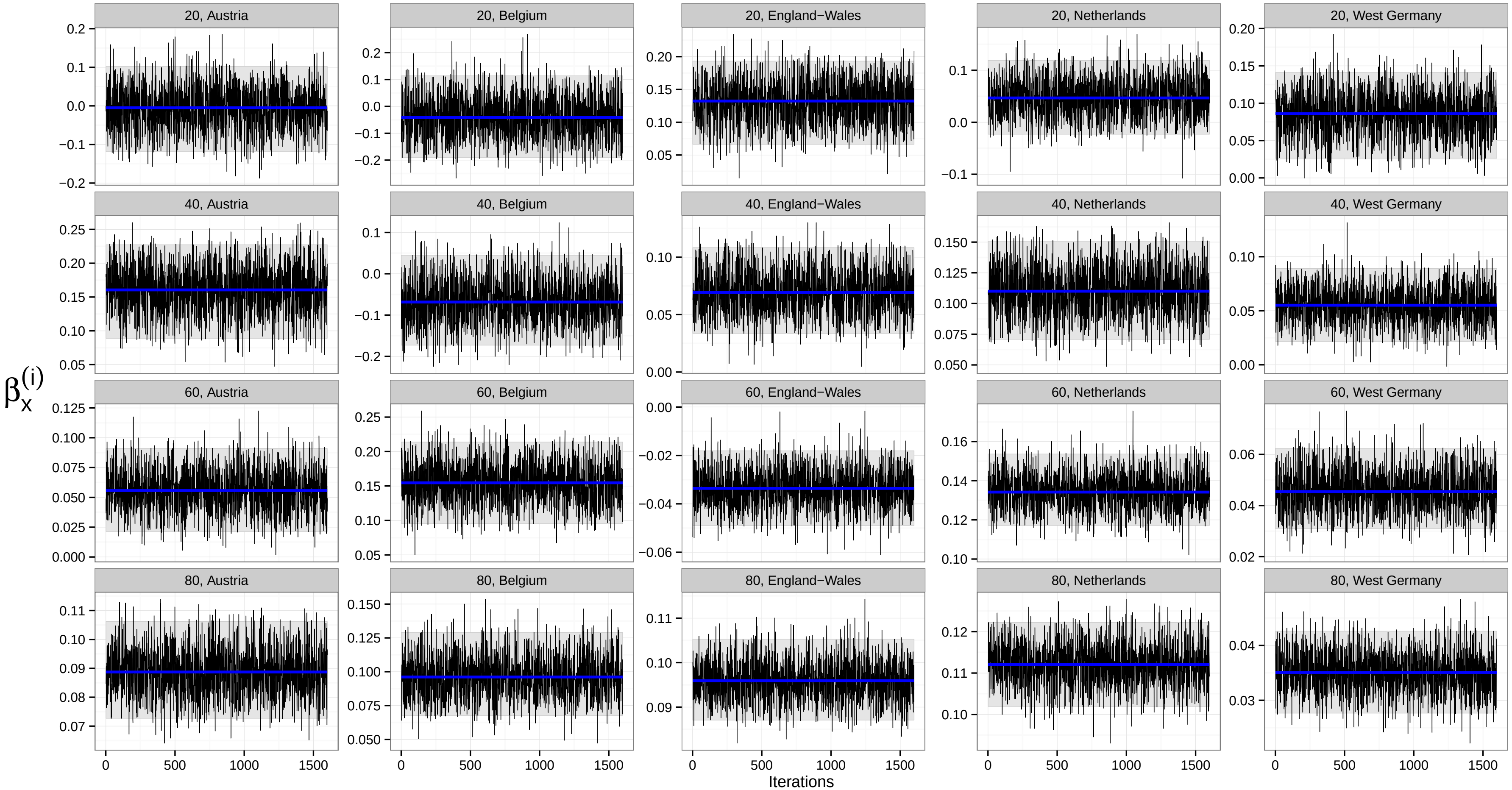


Figure 20: Model LL: trace plots for parameters $\beta_x^{(i)}$ at ages 20, 40, 60 and 80: female, period 1975-2009 and ages 0-89.

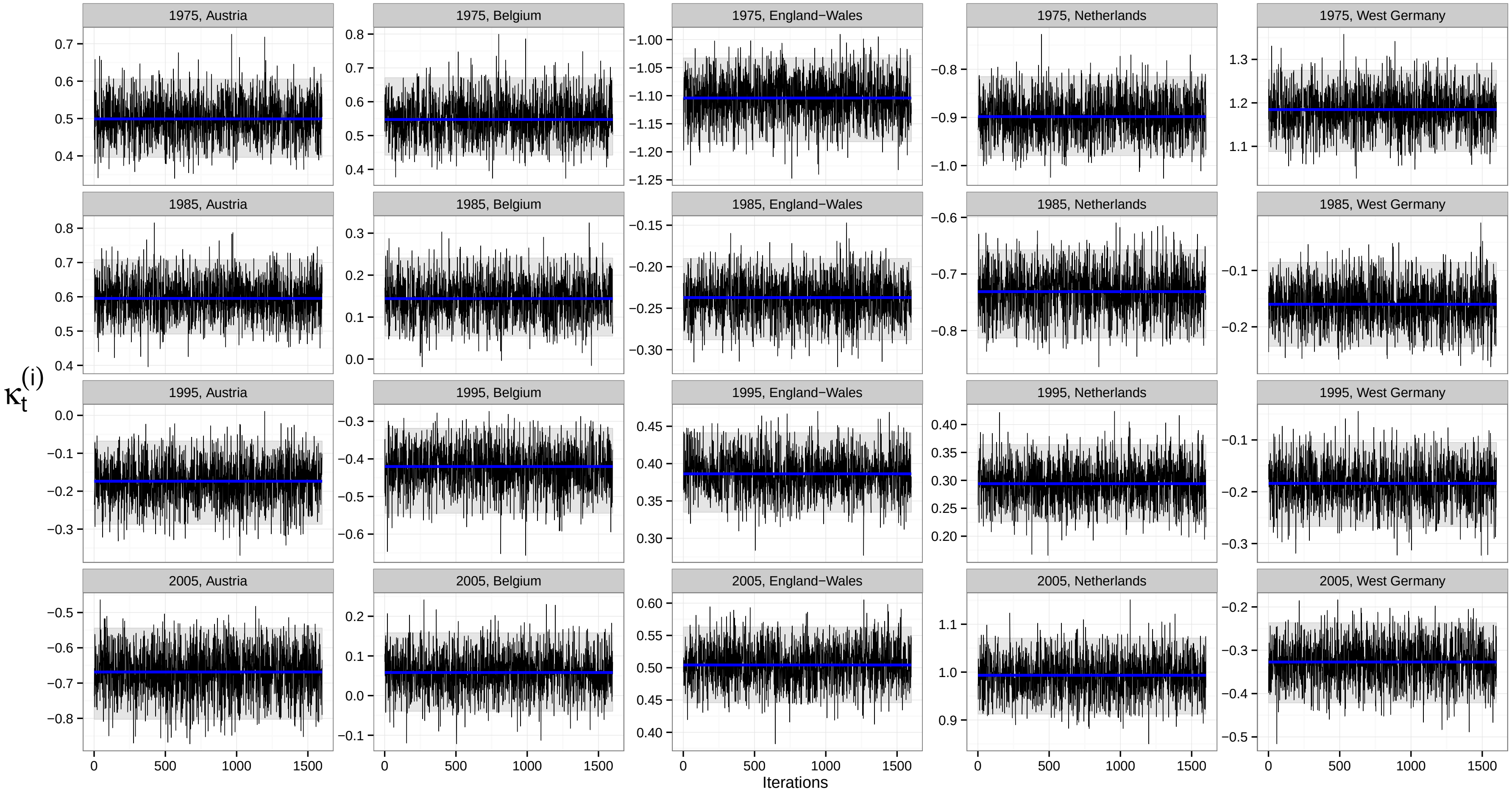


Figure 21: Model LL: trace plots for parameters $\kappa_t^{(i)}$ at years 1975, 1985, 1995 and 2005: female, period 1975-2009 and ages 0-89.

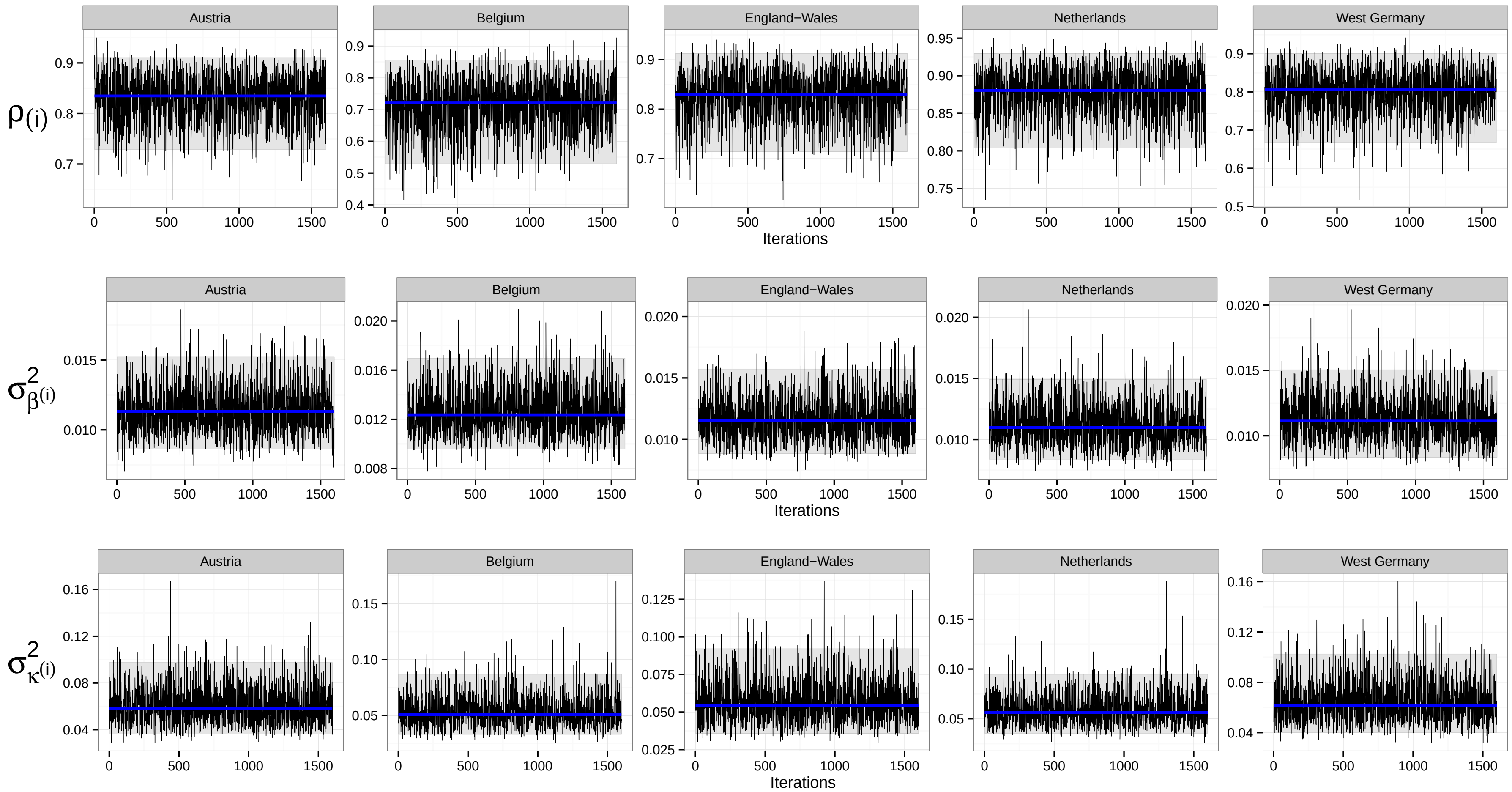


Figure 22: Model LL: trace plots for hyper-parameters $\rho_{(i)}$, $\sigma^2_{\beta(i)}$ and $\sigma^2_{\kappa(i)}$: female, period 1975-2009 and ages 0-89.

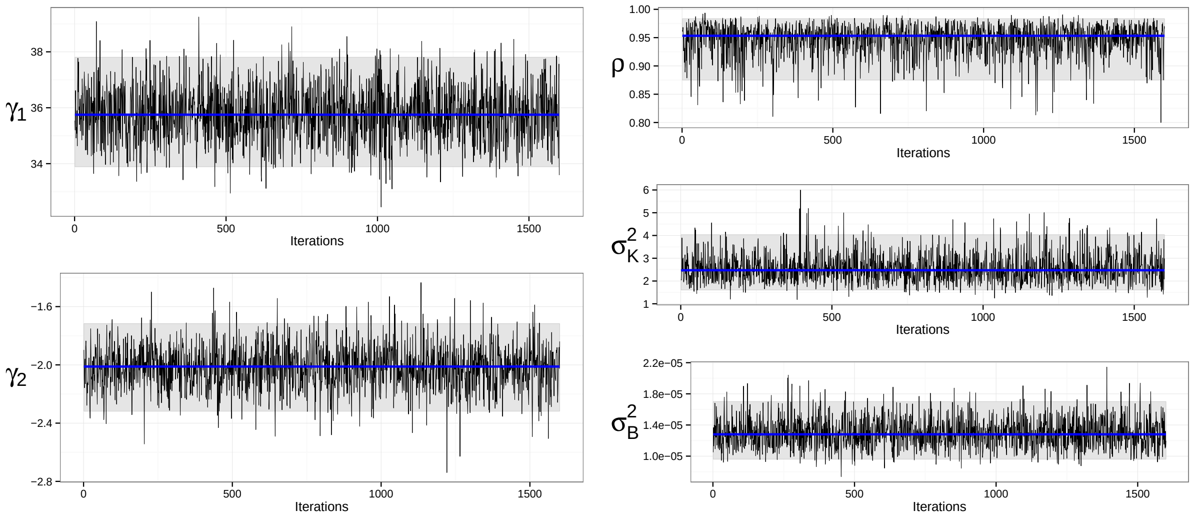


Figure 23: Model LL: trace plots for γ_1 (top left), γ_2 (bottom left) and hyper-parameters ρ (top right), σ_K^2 (middle right) σ_K^2 (bottom right): female, period 1975-2009 and ages 0-89.

References

Antonio, K., Bardoutsos, A., and Ouburg, W. (2015). Bayesian poisson log-bilinear models for mortality projections with multiple populations. *submitted to European Actuarial Journal, KU Leuven-AFI internal report 1599*.

Czado, C., Delwarde, A., and Denuit, M. (2005). Bayesian Poisson log–bilinear mortality projections. *Insurance: Mathematics and Economics*, 36:260–284.

FACULTY OF ECONOMICS AND BUSINESS
Naamsestraat 69 bus 3500
3000 LEUVEN, BELGIË
tel. + 32 16 32 66 12
fax + 32 16 32 67 91
info@econ.kuleuven.be
www.econ.kuleuven.be